ISSN
Print: 2695-2132 | Online: 2695-2118

An Optimal Block Backward Differentiation Formulae with Diagonalizations for Solving System of First Order Stiff IVPs

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Abstract

In this research, a new diagonally implicit block backward differentiation formula with optimal accuracy for solving stiff IVP of ODEs is developed. In the proposed schemes, three approximate solution values are computed concurrently at each integration step. The order and stability properties of the schemes are evaluated and it is found to be of order 5, Zero and A-Stable; capable of integrating Stiff IVP. Some Samples of first order stiff IVPs are computed, the performance of the methods are presented. The results are compared with some existing methods. The computed results with the plotted graphs have shown that the newly scheme possess an optimal accuracy in the scale error over the other methods compared. The proposed scheme is recommended for solving first order stiff IVP ODEs.

***Keywords**: A-Stable, Block, Implicit, IVPs, Points, Ordinary Differential Equation*

Introduction

In engineering and sciences, problems are modelled into different forms of special differential equations in some cases stiff ordinary differential equation, and to obtain their analytic and/or numerical solutions. Stiff equation is found usually to deviate from analytic solution, numerical approximate solution are often considered where the exact solution seen not feasible or highly complex to obtain. Most of the Problems in vibrations, chemical reactions, kinetics and electrical circuits are modelled into stiff IVPs. Many scholars have formulated different schemes with the aim of obtaining analytic and/or numerical solutions of the modelled Stiff IVPs, because many stiff problem cannot be solved analytically and also some numerical scheme like explicit method cannot handle stiff Problems effectively. Preferences are given to approximate numerical solution using implicit or partially implicit schemes as it can solve any form stiff IVP of ODEs. For that, approximate result are always in consideration in case of stiff IVP. The concern is to get an accurate scheme that would solve any stiff IVPs of ODE with minimum error and computation time. The early work in BDF came from [1, 2, 3], the block that compute more solution values at a time per integration step can be found in [4, 5, 6, 7]. Different BBDF method with good stability properties and various degree of accuracy include [8, 9, 10, 11]. Hybrid aspect of the BBDF also display very good accuracy and execution time [12,13,14,15,16,17,18,19,20,21,22,23]. The above stated scheme possessed different strenth in the accuracy and computational time.

This study considers developing a new block of fifth order diagonally implicit of backward differentiation formula for solving stiff IVP of ODEs of the form

$$y' = f(x,\ \hat{Y}), \qquad \hat{Y}(a) = b, \qquad a \le x \le b \qquad (1)$$

Where $\hat{Y} = (y_1, y_2, y_3, \dots \dots \dots y_n)$ and f is continuous within the interval of integration [a, b]. We assume that f satisfies Lipchitz condition which guarantees the existence and uniqueness of solution of eqn. (1)

Method Formulation

In this section, three approximate solution values y_{n+1} , y_{n+2} and y_{n+3} in a block simultaneously, are derived. The formulae are computed using three back values of y_{n-2} , y_{n-1} and y_n with constant step size h. The proposed method, An Improved Diagonally Implicit BBDF with Optimal accuracy For Solving System of First Order Stiff IVP derived using Taylor's series expansion about x_n will takes the form

$$\sum_{j=0}^{2+k} \alpha_{j,i} y_{n+j-2} = h\beta_{k,i}[f_{n+k} + f_{n+k+1}] \qquad k = 1, 2, 3 \qquad (2)$$

According to [2], the linear operator L_i associated with first, second and third point of the method is defined as follows:

$$\left.\begin{array}{r} L_1[y(x_n), h)]: \alpha_{0,1}\, y_{n-2} + \alpha_{1,1}\, y_{n-1} + \alpha_{2,1}\, y_n + \alpha_{3,1}\, y_{n+1} - h\beta_{1,1}[f_{n+1} + f_n] = 0 \\ L_2[y(x_n), h)]: \alpha_{0,2}\, y_{n-2} + \alpha_{1,2}\, y_{n-1} + \alpha_{2,2}\, y_n + \alpha_{3,2}\, y_{n+1} + \alpha_{4,2}\, y_{n+2} + - h\beta_{2,2}[f_{n+2} + f_{n+1}] = 0 \\ L_3[y(x_n), h)]: \alpha_{0,3}\, y_{n-2} + \alpha_{1,3}\, y_{n-1} + \alpha_{2,3}\, y_n + \alpha_{3,3}\, y_{n+1} + \alpha_{4,3}\, y_{n+2} + \alpha_{5,3}\, y_{n+3} - h\beta_{3,3}[f_{n+3} + f_{n+2}] = 0 \end{array}\right\}$$

(3)

Consider the following value of k & $i's$ value in (3) for the cases below:

FOR CASE 1, 2 & 3 as in $k = i = 1$, $k = i = 2$, & $k = i = 3$ for the First, Second, & Third point respectively, with the associated operator (L_1 , L_2 ,& L_3) related to (3) written as

$$\left.\begin{array}{c} \alpha_{0,1}y(x_n - 2h) + \alpha_{1,1}y(x_n - h) + \alpha_{2,1}y(x_n) + \alpha_{3,1}y(x_n + h) - h\beta_{1,1}[f_{n+1} + f_n] = 0 \\ \alpha_{0,2}y(x_n - 2h) + \alpha_{1,2}y(x_n - h) + \alpha_{2,2}y(x_n) + \alpha_{3,2}y(x_n + h) + \alpha_{4,2}y(x_n + 2h) - \\ h_{2,2}[f_{n+2} + f_{n+1}] = 0 \\ \alpha_{0,3}y(x_n - 2h) + \alpha_{1,3}y(x_n - h) + \alpha_{2,3}y(x_n) + \alpha_{3,3}y(x_n + h) + \alpha_{4,3}y(x_n + 2h) + \alpha_{5,3}y(x_n + 3h) \\ -h\beta_{3,3}[f_{n+3} + f_{n+2}]] = 0 \end{array}\right\}$$

(4)

Expanding $(x_n - 2h)$, $y(x_n - h)$, $y(x_n)$, $y(x_n + h)$, $y(x_n + 2h)$, $y(x_n + 3h)$, $f(x_n + h), f(x_n + 2h), f(x_n + 3h)$ $f(x_n)$ in (4) with a Taylor's series expansion about x_n and collect the like terms gives

$$\left.\begin{array}{r} C_{0,1}y(x_n) + C_{1,1}hy'(x_n) + C_{2,1}h^2y''(x_n) + \cdots = 0 \\ C_{0,2}y(x_n) + C_{1,2}hy'(x_n) + C_{2,2}h^2y''(x_n) + C_{3,2}h^3y'''(x_n) + \cdots = 0 \\ C_{0,3}y(x_n) + C_{1,3,}hy'(x_n) + C_{2,3}h^2y''(x_n) + C_{3,3}h^3y'''(x_n) + C_{4,3}h^4y'^{v}(x_n) + \cdots = 0 \end{array}\right\} \qquad (5)$$

Where (5) is evaluated respectively as follows

$$\left.\begin{array}{r} C_{0,1} = \alpha_{0,1} + \alpha_{1,1} + \alpha_{2,1} + \alpha_{3,1} = 0 \\ C_{1,1} = -2\alpha_{0,1} - \alpha_{1,1} + \alpha_{3,1} - 2\beta_{1,1} = 0 \\ C_{2,1} = 2\alpha_{0,1} + \frac{1}{2}\alpha_{1,1} + \frac{1}{2}\alpha_{3,1} - \beta_{1,1} = 0 \\ C_{3,1} = -\frac{4}{3}\alpha_{0,1} - \frac{1}{6}\alpha_{1,1} + \frac{1}{6}\alpha_{3,1} - \frac{1}{2}\beta_{1,1} = 0 \end{array}\right\} \qquad (6)$$

$$\left.\begin{aligned} C_{0,2} &= \alpha_{0,2}+\alpha_{1,2}+\alpha_{2,2}+\alpha_{3,2}+\alpha_{4,2}=0 \\ C_{1,2} &= -2\alpha_{0,2}-\alpha_{1,2}+\alpha_{3,2}+2\alpha_{4,2}\pm 2\beta_{2,2}=0 \\ C_{2,2} &= 2\alpha_{0,1}+\frac{1}{2}\alpha_{1,2}+\frac{1}{2}\alpha_{3,2}+2\alpha_{4,2}-3\beta_{2,2}=0 \\ C_{3,2} &= -\frac{4}{3}\alpha_{0,2}-\frac{1}{6}\alpha_{1,2}+\frac{1}{6}\alpha_{3,2}+\frac{4}{3}\alpha_{4,2}-\frac{5}{2}\beta_{2,2}=0 \\ C_{4,2} &= \frac{2}{3}\alpha_{0,2}+\frac{1}{24}\alpha_{1,2}+\frac{1}{24}\alpha_{3,2}+\frac{2}{3}\alpha_{4,2}+-\frac{3}{2}\beta_{2,2}=0 \end{aligned}\right\}$$
(7)

&

$$\left.\begin{aligned} C_{0,3} &= \alpha_{0,3}+\alpha_{1,3}+\alpha_{2,3}+\alpha_{3,3}+\alpha_{4,3}+\alpha_{5,3}=0 \\ C_{1,3} &= -2\alpha_{0,3}-\alpha_{1,3}+\alpha_{3,3}+2\alpha_{4,3}+3\alpha_{5,3}-2\beta_{3,3}=0 \\ C_{2,3} &= 2\alpha_{0,3}+\frac{1}{2}\alpha_{1,3}+\frac{1}{2}\alpha_{3,3}+2\alpha_{4,3}+\frac{9}{2}\alpha_{5,1}-5\beta_{3,3}=0 \\ C_{3,3} &= -\frac{4}{3}\alpha_{0,3}-\frac{1}{6}\alpha_{1,3}+\frac{1}{6}\alpha_{3,3}+\frac{4}{3}\alpha_{4,3}+\frac{9}{2}\alpha_{5,3}-\frac{13}{2}\beta_{3,3}=0 \\ C_{4,3} &= \frac{2}{3}\alpha_{0,3}+\frac{1}{24}\alpha_{1,3}+\frac{1}{24}\alpha_{3,3}++\frac{2}{3}\alpha_{4,3}++\frac{81}{24}\alpha_{5,3}-\frac{35}{6}\beta_{3,3}=0 \\ C_{5,3} &= -\frac{4}{15}\alpha_{0,3}-\frac{1}{120}\alpha_{1,3}+\frac{1}{120}\alpha_{3,3}+\frac{4}{15}\alpha_{4,3}+\frac{81}{40}\alpha_{5,3}-\frac{97}{24}\beta_{3,3}=0 \end{aligned}\right\} \quad (8)$$

Normalizing the coefficients $\alpha_{\frac{3}{2},\frac{1}{2}}$, $\alpha_{2,1}$, $\alpha_{\frac{5}{2},\frac{3}{2}}$ & $\alpha_{3,2}$ of $y_{n+\frac{1}{2}}$, y_{n+1}, $y_{n+\frac{3}{2}}$ & y_{n+2} respectively to 1, solving equation (6), (7), & (8) with the aids of Maple Software for the values of $\alpha_{j,i's}$ and $\beta_{j,i's}$and Substituting the values in (4) gives the first, second, third & fourth point as

$$\left.\begin{aligned} y_{n+1} &= -\frac{1}{13}y_{n-2}-\frac{1}{13}y_{n-1}+\frac{15}{13}y_n+\frac{6}{13}hf_{n+1}+\frac{6}{13}hf_n \\ y_{n+2} &= -\frac{19}{84}y_{n-2}+\frac{83}{28}y_{n-1}-\frac{155}{28}y_n+\frac{319}{84}y_{n+1}+\frac{5}{14}hf_{n+2}+\frac{5}{14}hf_{n+1} \\ y_{n+3} &= \frac{9}{149}y_{n-2}-\frac{55}{149}y_{n-1}+\frac{140}{149}y_n-\frac{180}{149}y_{n+1}+\frac{235}{149}y_{n+2}+\frac{60}{149}hf_{n+3}+\frac{60}{149}hf_{n+2} \end{aligned}\right\} \quad (9)$$

Analysis of the Method

In this section, order and Stability properties of the proposed method (9) will be analysed.

Order of the Method

In this section, the order of the method (9) will be derived. The Method can be transform to a general matrix form as

$$\sum_{j=0}^{1} C_j^* Y_{m+j-1} = h\sum_{j=0}^{1} D_j^* Y_{m+j-1}, \quad (10)$$

Where C & D are constant coefficient matrices of the method.

(10) is equivalent to the following form

$$\left.\begin{aligned} \frac{1}{13}y_{n-2}+\frac{1}{13}y_{n-1}-\frac{15}{13}y_n+y_{n+1} &= \frac{6}{13}hf_{n+1}+\frac{6}{13}hf_n \\ \frac{19}{84}y_{n-2}-\frac{83}{28}y_{n-1}-\frac{1}{55}y_n-\frac{319}{84}y_{n+1}+y_{n+2} &= \frac{5}{14}hf_{n+2}+\frac{5}{14}hf_{n+1} \\ -\frac{9}{149}y_{n-2}+\frac{55}{149}y_{n-1}-\frac{140}{149}y_n+\frac{180}{149}y_{n+1}-\frac{235}{149}y_{n+2}+y_{n+3} &= \frac{60}{149}hf_{n+3}+\frac{60}{149}hf_{n+2} \end{aligned}\right\}$$
(11)

Also (11) can be written as

$$\begin{bmatrix} -\frac{1}{13} & \frac{3}{13} & -\frac{15}{13} \\ \frac{19}{84} & -\frac{83}{28} & \frac{155}{28} \\ -\frac{9}{149} & \frac{55}{149} & \frac{-140}{149} \end{bmatrix}\begin{bmatrix} y_{n-2} \\ y_{n-1} \\ y_n \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ -\frac{319}{84} & 1 & 0 \\ \frac{180}{149} & -\frac{235}{149} & 1 \end{bmatrix}\begin{bmatrix} y_{n+1} \\ y_{n+2} \\ y_{n+3} \end{bmatrix} = h\begin{bmatrix} 0 & 0 & \frac{6}{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} f_{n-2} \\ f_{n-1} \\ f_n \end{bmatrix} +$$

$$h\begin{bmatrix} \frac{6}{13} & 0 & 0 \\ \frac{5}{14} & \frac{5}{14} & 0 \\ 0 & \frac{60}{149} & \frac{60}{149} \end{bmatrix}\begin{bmatrix} f_{n+1} \\ f_{n+2} \\ f_{n+3} \end{bmatrix}$$

(12)

where

$$C_0 = \begin{bmatrix} \frac{1}{13} \\ \frac{19}{84} \\ -\frac{9}{149} \end{bmatrix} \quad C_1 = \begin{bmatrix} \frac{1}{13} \\ -\frac{83}{28} \\ \frac{55}{149} \end{bmatrix} \quad C_2 = \begin{bmatrix} \frac{15}{13} \\ \frac{155}{28} \\ -\frac{140}{149} \end{bmatrix} \quad C_3 = \begin{bmatrix} 1 \\ -\frac{319}{84} \\ \frac{180}{149} \end{bmatrix} \quad C_4 = \begin{bmatrix} 0 \\ 1 \\ -\frac{235}{149} \end{bmatrix} \quad C_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$D_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad D_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad D_2 = \begin{bmatrix} \frac{6}{13} \\ 0 \\ 0 \end{bmatrix} \quad D_3 = \begin{bmatrix} \frac{6}{13} \\ \frac{5}{14} \\ 0 \end{bmatrix} \quad D_4 = \begin{bmatrix} 0 \\ \frac{5}{14} \\ \frac{60}{149} \end{bmatrix} \quad D_5 = \begin{bmatrix} 0 \\ 0 \\ \frac{60}{149} \end{bmatrix}$$

Definition 3.1.1 According to [16], the order of the block method (10) and its associated linear operator are given by

$$L[y(x); h] = \sum_{j=0}^{7}[C_j y(x + jh)] - h\sum_{j=0}^{7}[D_j y'(x + jh)] \qquad (13)$$

Where p is unique integer such that

$E_q = 0, \ q = 0,1, \ldots \ p$ and $E_{p+1} \neq 0$, where the E_q are constant Matrix with

$$E_0 = \sum_{J=0}^{5} C_J = 0$$

$$E_1 = \sum_{J=0}^{5}[jC_j - 2D_j] = 0$$

$$E_2 = \sum_{J=0}^{5}\left[\frac{1}{2!}j^2C_j - 2jD_j\right] = 0$$

$$E_3 = \sum_{J=0}^{5}\left[\frac{1}{3!}j^3C_j - 2\frac{1}{2!}j^2D_j\right] = 0$$

$$E_4 = \sum_{J=0}^{5}\left[\frac{1}{4!}j^4C_j - 2\frac{1}{3!}j^3D_j\right] = 0$$

$$E_5 = \sum_{J=0}^{5}\left[\frac{1}{5!}j^5C_j - 2\frac{1}{4!}j^4D_j\right] = 0$$

$$E_6 = \sum_{J=0}^{5}\left[\frac{1}{6!}j^6C_j - 2\frac{1}{5!}j^5D_j\right] = \begin{pmatrix} -\frac{121}{9353} \\ -\frac{221}{5065} \\ -\frac{274}{7385} \\ -\frac{192}{5123} \end{pmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence, the method is of order 5, with error constant $E_6 = \begin{pmatrix} -\frac{121}{9353} \\ -\frac{221}{5065} \\ -\frac{274}{7385} \\ -\frac{192}{5123} \end{pmatrix}$ (14)

Stability Analysis of the Method

In this section, we investigate the Zero and A- Stability property of the proposed method (9).

Definition 3.2.1 A linear multistep method is said to be zero stable if no root of the first characteristics polynomial has modulus greater than one and that any root with modulus one is simple [20]

Definition 3.2.2 A linear multistep Method is said to be an A-stable method if its stability region covers the entire negative half-plane [20]

The stability of the method (10-11) can be obtains by applying the standard test equation of the form

$$y' = \lambda y \qquad \lambda \; is \; a \; complex \; number, \qquad Re(\lambda) < 0 \tag{15}$$

Substitute (15) into (9) get the following solutions

$$\left.\begin{aligned} y_{n+1} &= -\frac{1}{13}y_{n-2} - \frac{1}{13}y_{n-1} + \frac{15}{13}y_n + \frac{6}{13}h\lambda y_{n+1} + \frac{6}{13}h\lambda y_n \\ y_{n+2} &= -\frac{19}{84}y_{n-2} + \frac{83}{28}y_{n-1} - \frac{155}{28}y_n + \frac{319}{84}y_{n+1} + \frac{5}{14}h\lambda y_{n+2} + \frac{5}{14}h\lambda y_{n+1} \\ y_{n+3} &= \frac{9}{149}y_{n-2} - \frac{55}{149}y_{n-1} + \frac{140}{149}y_n - \frac{180}{149}y_{n+1} + \frac{235}{149}y_{n+2} + \frac{60}{149}h\lambda y_{n+3} + \frac{60}{149}h\lambda y_{n+2} \end{aligned}\right\}$$

(16)

(16) Can also be written as

$$\begin{bmatrix} 1-\frac{6}{13}h\lambda & 0 & 0 \\ -\frac{319}{84} & 1-\frac{5}{14}h\lambda & 0 \\ \frac{180}{149} & -\frac{235}{149} & 1-\frac{60}{149}h\lambda \end{bmatrix} \begin{bmatrix} y_{n+1} \\ y_{n+2} \\ y_{n+3} \end{bmatrix} = \begin{bmatrix} \frac{1}{13} & \frac{1}{13} & \frac{15}{13}+\frac{6}{13}h\lambda \\ -\frac{19}{84} & \frac{83}{28} & -\frac{155}{28} \\ \frac{9}{149} & -\frac{55}{149} & \frac{140}{149} \end{bmatrix} \begin{bmatrix} y_{n-2} \\ y_{n-1} \\ y_n \end{bmatrix}$$

(17)

From (17) it is given that

$$AY_m = BY_{m-1} \quad (19)$$

If m is the number of block and r is the number of points in the block, then n = mr

Here, r = 2 and n = 2m. It follows that

$$Y_m = \begin{bmatrix} y_{2m+1} \\ y_{2m+2} \\ y_{2m+3} \end{bmatrix} = \begin{bmatrix} y_{n+1} \\ y_{n+2} \\ y_{n+3} \end{bmatrix}, \; Y_{m-1} = \begin{bmatrix} y_{n-2} \\ y_{n-1} \\ y_n \end{bmatrix} \quad (20)$$

And the coefficient matrices are given as

$$A = \begin{bmatrix} 1-\frac{6}{13}h\lambda & 0 & 0 \\ -\frac{319}{84} & 1-\frac{5}{14}h\lambda & 0 \\ \frac{180}{149} & -\frac{235}{149} & 1-\frac{60}{149}h\lambda \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{13} & \frac{1}{13} & \frac{15}{13}+\frac{6}{13}h\lambda \\ -\frac{19}{84} & \frac{83}{28} & -\frac{155}{28} \\ \frac{9}{149} & -\frac{55}{149} & \frac{140}{149} \end{bmatrix}$$

(21)

The stability polynomial of the proposed method was computed with the aid of Maple Software and the result is found to be

$$det(At - B) = -\frac{15381}{13559}t + \frac{46}{1043}h + \frac{340}{13559} + t^3 - \frac{34571}{27118}th + \frac{114}{1043}t^2 - \frac{3964}{1043}t^2h - \frac{42165}{13559}t^2h^2 - \frac{33121}{27118}t^3h + \frac{45}{91}t^3h^2 - \frac{900}{13559}t^3h^3 + \frac{1530}{13559}th^2 - \frac{900}{13559}t^2h^3 \quad (22)$$

$$R(t,0) = -\frac{15381}{13559}t + \frac{340}{13559} + t^3 + \frac{114}{1043}t^2 \quad (23)$$

$t = 0.0221621149, 1 \;\&\; 1$

3.3 A- Stability of the Proposed Method

In this section, the region for the absolute stability of the proposed method will be plotted, by considering the stability polynomials (20). The set of point defined by $t = e^{i\theta}$, $0 \le \theta \le 2\pi$ describes the boundary of the stability region. The following stability region was the complex plot of the proposed method with the aid of Maple Software.

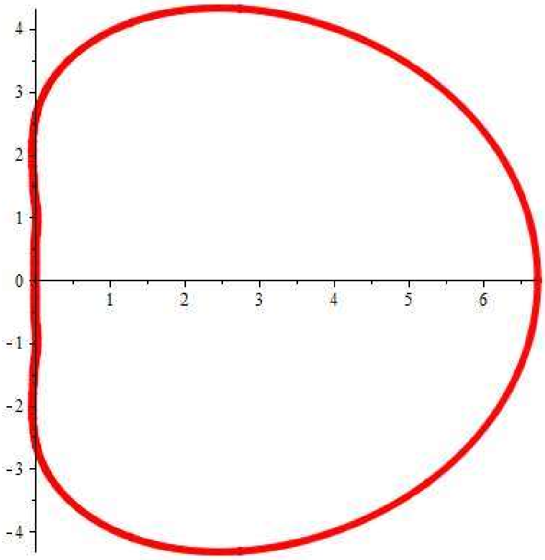

Figure 2: A-Stability region of the Proposed Method (RDOBBDF)

1.1 Test Problems

To validate the method developed, (3DIBBDF), a code in ‘C’ (programming Language) would be used to solve the following stiff IVPs below

Table 1 Sample of First Order Initial Value Problem of Stiff ODEs

$$Problem 1$$

$$\left.\begin{aligned} \boldsymbol{y_1'} &= 998y_1 + 1998y_2 \quad y_1(0) = 1 \\ \boldsymbol{y_2'} &= -\boldsymbol{999}y_1 - 1999y_2 \quad y_2\ (0) = 0 \end{aligned}\right\} 0 \leq x \leq 20$$

$$exact\ solution$$

$$\boldsymbol{y_1(x) = 2e^{-x} - 2e^{-1000x}}$$

$$\boldsymbol{y_2(x) = -e^{-x} - e^{-1000x}}$$

$$\lambda_1 = -1, \quad \lambda_2 = -1000$$

source (Abasi *et al 2014*)

$$Problem 2$$

$$\left.\begin{aligned} \boldsymbol{y_1'} &= 198y_1 + 199y_2 \quad y_1(0) = 1 \\ \boldsymbol{y_2'} &= -\boldsymbol{999}y_1 - 1999y_2 \quad y_2\ (0) = -1 \end{aligned}\right\} 0 \leq x \leq 10$$

$$exact\ solution$$

$$\boldsymbol{y_1(x) = e^{-x}}$$

$$\boldsymbol{y_2(x) = -e^{-x}}$$

Source (Ibrahim *et al* 2007)

Result and Discussions

The sample problems presented in this paper are solved using the proposed methods. The result of the tested problems are tabulated and compared with the existing ones. The graphs highlighting the performance of these methods are plotted. The acronyms below are used in the tables.

H= step-size;

MHTD =Method

MAX-ERR = Maximum Error;

3ESBBDF = Extended 3-Point Super class of Block Backward Differentiation formula for Solving Stiff Initial Value Problems.

3DIBBDF = an improved Diagonally Implicit of Block Backward Differentiation Formula with optimal accuracy for Solving Stiff IVP of ODEs

3BBDF = Implicit r-point block backward differentiation formula for solving first-order stiff ODEs

ODEs23 = MATLAB's Stiff numerical solvers for ODEs

NVSBBDF=New variable step size block backward differential formula

2BBDF = Implicit r-point block backward differentiation formula for solving first-order stiff ODEs

Table of maximum errors of numerical solutions for problem 1

Numerical Result for Problem 1			
H	**MTHD**	**NS**	**MAX-ERR**
10^{-2}	3BBDF NVSBBDF Ode23 3DIBBDF	118 49 23 888	$1.02670(-4)$ $3.53673(-4)$ $7.30000(-3)$ $7.20477(-3)$
10^{-4}	3BBDF NVSBBDF Ode23 3DIBBDF	144 74 68 88888	$7.08820(-6)$ $1.59269(-5)$ $3.68370(-4)$ $7.21273(-7)$
10^{-6}	3BBDF NVSBBDF Ode23 3DIBBDF	171 118 288 8888888	$1.10060(-8)$ $8.11971(-7)$ $1.70390(-5)$ 7.20514 (-11)

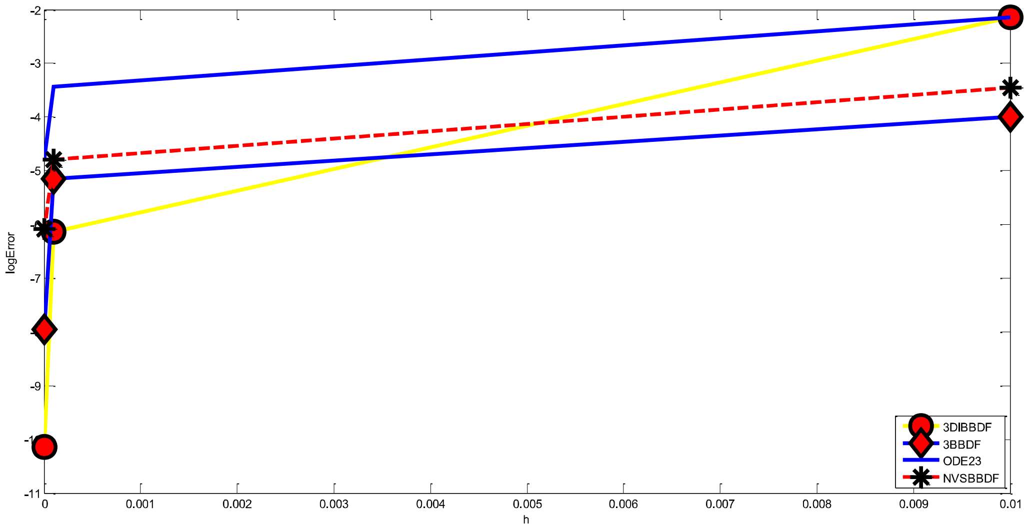

Figure 2 Graph of $\text{Log}_{10}(MAXE)$ against the step size h for problem 1

Table of maximum errors of numerical solutions for problem 2

h	METHOD	NS	MAXE
10^{-2}	2BBDF	500	7.18323 (-3)
	3BBDF	333	1.07308(-2)
	3ESBBDF	333	1.83217(-4)
	3DIBBDF	335	1.50983(-3)
10^{-3}	2BBDF	5,000	7.34012(-4)
	3BBDF	3,333	1.10060(-3)
	3ESBBDF	3,333	8.05338(-6)
	3DIBBDF	3,334	1.67758(-5)
10^{-4}	2BBDF	50,000	7.35584(-5)
	3BBDF	33,333	1.10333(-4)
	3ESBBDF	33,333	1.26692(-8)
	3DIBBDF	33,334	1.52508(-7)
10^{-5}	2BBDF	500,000	7.35741(-6)
	3BBDF	333,333	1.10361(-5)
	3ESBBDF	333,333	1.32740(-10)
	3DIBBDF	333,334	1.52558(-7)
10^{-6}	2BBDF	5,000,000	7.35747(-7)
	3BBDF	3,333,333	1.10363(-6)
	3ESBBDF	3,333,333	1.33362(-12)
	3DIBBDF	3,333,3334	1.51063(-11)

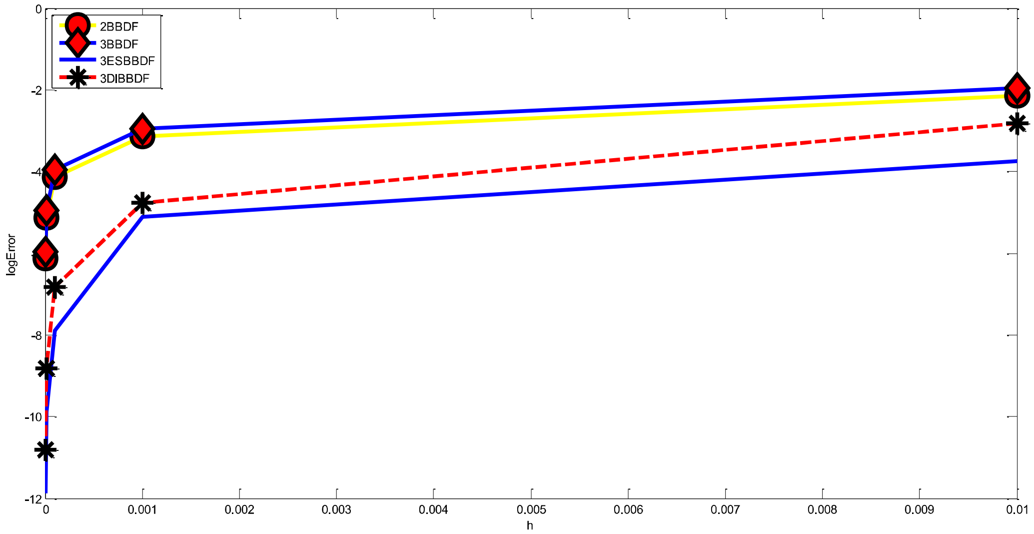

Figure 3 Graph of $\text{Log}_{10}(MAXE)$ against the step size h for problem 2

From table 2 and 3 comprising examples 1 and 2 it shows that the new proposed scheme, **3DIBBDF** outperformed the 3BBDF, ODEs23 and NVSBBDF in terms accuracy in problems 1. While, 3BBDF has a good accuracy over NVSBBDF and ODEs23 in problems 1. In table 2 consisting example 2, the 3DIBBDF and 3ESBBDF competes closely in terms of accuracy of the scale errors, with 3ESBBDF has a competing advantage over the method.

To further depicts the visible performance of the schemes, the graphs of *Log10(MAXE)* against the step size *h* for all the examples considered are plotted (Using Matlab) in figure 2 and 3. From the figures the accuracy of the method compared are highlighted clearly. Hence, the proposed newly developed method recommended to be an alternative solver for a first order stiff IVPs.

Conclusion

A new diagonally implicit scheme is developed, the method is block; can generate three approximate solution values at a time per iteration. The properties of the proposed scheme are investigated, the method is of order five, zero and A – Stable, then it can handle stiff IVPs. Sampled problems solved validated superiority of the method in terms of accuracy of the scale error over some of the scheme compared. Hence, the newly proposed method can be used in solving stiff IVPs.

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